S_3-flavour symmetry as realized in lepton flavour violating processes

2008 J. Phys. A: Math. Theor. 41 304035

(http://iopscience.iop.org/1751-8121/41/30/304035)

J. Phys. A: Math. Theor. **41** (2008) 304035 (15pp) doi:10.1088/1751-8113/41/30/304035

S_3-flavour symmetry as realized in lepton flavour violating processes

A Mondragón, M Mondragón and E Peinado

Instituto de Física, Universidad Nacional Autónoma de México, Apdo. Postal 20-364, 01000 México DF, Mexico

E-mail: mondra@fisica.unam.mx, myriam@fisica.unam.mx and eduardo@fisica.unam.mx

Received 25 October 2007, in final form 10 December 2007
Published 15 July 2008
Online at stacks.iop.org/JPhysA/41/304035

Abstract

A variety of lepton flavour violating effects related to the recent discovery of neutrino oscillations and mixings is here systematically discussed in terms of an S_3-flavour permutational symmetry. After a brief review of some relevant results on lepton masses and mixings, that had been derived in the framework of a minimal S_3-invariant extension of the Standard Model, we derive explicit analytical expressions for the matrices of the Yukawa couplings and compute the branching ratios of some selected flavour-changing neutral current (FCNC) processes as well as the contribution of the exchange of neutral flavour-changing scalars to the anomaly of the muon's magnetic moment as functions of the masses of the charged leptons and the neutral Higgs bosons. We find that the $S_3 \times Z_2$ flavour symmetry and the strong mass hierarchy of the charged leptons strongly suppress the FCNC processes in the leptonic sector well below the present experimental upper bounds by many orders of magnitude. The contribution of FCNC to the anomaly of the muon's magnetic moment is small but non-negligible.

PACS numbers: 11.30.Hv, 14.60.Pq, 14.60.St, 14.80.Cp, 12.15.Ff, 12.15.Mm

1. Introduction

Neutrino oscillation observations and experiments, made in the past 9 years, have allowed the determination of the differences of the squared neutrino masses and the mixing angles in the leptonic sector [1–19]. The discovery that neutrinos have non-vanishing masses and mix among themselves much like the quarks do provides the first conclusive evidence of new physics beyond the Standard Model. This important discovery also brought out very forcefully the need of extending the Standard Model to accommodate in the theory the new data on neutrino physics in a coherent way, free of contradictions and without spoiling the Standard Model's many phenomenological successes.

In the Standard Model, the Higgs and Yukawa sectors, which are responsible for the generation of the masses of quarks and charged leptons, do not give mass to the neutrinos. Furthermore, the Yukawa sector of the Standard Model already has too many parameters whose values can only be determined from experiment. These two facts point to the necessity and convenience of extending the Standard Model in order to make a unified and systematic treatment of the observed hierarchies of masses and mixings of all fermions as well as the presence or absence of CP violating phases in the mixing matrices. At the same time, we would also like to reduce drastically the number of free parameters in the theory. These two seemingly contradictory demands can be met by means of a flavour symmetry under which the families transform in a non-trivial fashion.

Recently, we argued that such a flavour symmetry unbroken at the Fermi scale is the permutational symmetry of three objects S_3 and introduced a minimal S_3-invariant extension of the Standard Model [20]. In this model, we imposed S_3 as a fundamental symmetry in the matter sector. This assumption led us necessarily to extend the concept of flavour and generations to the Higgs sector. Hence, going to the irreducible representations of S_3, we added to the Higgs $SU(2)_L$ doublet in the S_3-singlet representation two more Higgs $SU(2)_L$ doublets, which can only belong to the two components of the S_3-doublet representation, in this way all the matter fields in the minimal S_3-invariant extension of the Standard Model—Higgs, quark and lepton fields, including the right-handed neutrino fields—belong to the three-dimensional representation $\mathbf{1} \oplus \mathbf{2}$ of the permutational group S_3. The leptonic sector of the model was further constrained by an Abelian Z_2 symmetry. We found that the $S_3 \times Z_2$ symmetry predicts an almost maximal $\sin\theta_{23}$ and a very small value for $\sin\theta_{13}$ and an inverted mass hierarchy of the left-handed neutrinos in good agreement with experiment [20, 21]. More recently, we reparametrized the mass matrices of the charged leptons and neutrinos, previously derived in [20], in terms of their eigenvalues and computed the neutrino mixing matrix, V_{PMNS}, and the neutrino mixing angles and Majorana phases as functions of the masses of charged leptons and neutrinos. The numerical values of the reactor, θ_{13}, and atmospheric, θ_{23}, mixing angles are determined only by the masses of the charged leptons in very good agreement with experiment. The solar mixing angle, θ_{12}, is almost insensitive to the values of the masses of the charged leptons, but its experimental value allowed us to fix the scale and origin of the neutrino mass spectrum. We found that the theoretical neutrino mixing matrix V_{PMNS} is nearly tri-bimaximal in excellent agreement with the latest experimental values [22, 23].

The symmetry S_3 [24–33] and the symmetry product groups $S_3 \times S_3$ [33–36] and $S_3 \times S_3 \times S_3$ [37, 38] broken at the Fermi scale have been considered by many authors to explain successfully the hierarchical structure of quark masses and mixings in the Standard Model. Some other interesting models based on the S_3, S_4, A_4 and D_5 flavour symmetry groups, unbroken at the Fermi scale, have also been proposed [39–46]. Recent flavour symmetry models are reviewed in [47–50], see also the references therein.

In this paper, after a short, updated review of some relevant results on lepton masses and mixings, we had previously derived, we will discuss some other important flavour violating effects in the minimal S_3-invariant extension of the Standard Model. We will give exact explicit expressions for the matrices of the Yukawa couplings in the leptonic sector expressed as functions of the masses of charged leptons and neutral Higgs bosons. With the help of the Yukawa matrices we will compute the branching ratios of some selected FCNC processes and the contribution of the exchange of neutral flavour-changing scalars to the anomaly of the muon's magnetic moment. We find that the interplay of the $S_3 \times Z_2$ flavour symmetry and the strong mass hierarchy of charged leptons strongly suppress the FCNC processes in the leptonic sector well below the experimental upper bounds by many orders of magnitude. The contribution to the anomaly, a_μ, from FCNC is at most 6% of the discrepancy between the experimental value and the Standard Model prediction for a_μ, which is a small but not negligible contribution.

2. The minimal S_3-invariant extension of the Standard Model

In the Standard Model analogous fermions in different generations have identical couplings to all gauge bosons of the strong, weak and electromagnetic interactions. Prior to the introduction of the Higgs boson and mass terms, the Lagrangian is chiral and invariant with respect to permutations of the left and right fermionic fields.

The six possible permutations of three objects (f_1, f_2, f_3) are elements of the permutational group S_3. This is the discrete, non-Abelian group with the smallest number of elements. The three-dimensional real representation is not an irreducible representation of S_3. It can be decomposed into the direct sum of a doublet f_D and a singlet f_s, where

$$\begin{aligned} f_s &= \frac{1}{\sqrt{3}}(f_1 + f_2 + f_3), \\ f_D^T &= \left(\frac{1}{\sqrt{2}}(f_1 - f_2), \frac{1}{\sqrt{6}}(f_1 + f_2 - 2f_3)\right). \end{aligned} \tag{1}$$

The direct product of two doublets $\mathbf{p_D}^T = (p_{D1}, p_{D2})$ and $\mathbf{q_D}^T = (q_{D1}, q_{D2})$ may be decomposed into the direct sum of two singlets $\mathbf{r_s}$ and $\mathbf{r_{s'}}$ and one doublet $\mathbf{r_D}^T$ where

$$\mathbf{r_s} = p_{D1}q_{D1} + p_{D2}q_{D2}, \qquad \mathbf{r_{s'}} = p_{D1}q_{D2} - p_{D2}q_{D1}, \tag{2}$$

$$\mathbf{r_D}^T = (r_{D1}, r_{D2}) = (p_{D1}q_{D2} + p_{D2}q_{D1}, p_{D1}q_{D1} - p_{D2}q_{D2}). \tag{3}$$

The antisymmetric singlet $\mathbf{r_{s'}}$ is not invariant under S_3.

Since the Standard Model has only one Higgs $SU(2)_L$ doublet, which can only be an S_3 singlet, it can only give mass to the quark or charged lepton in the S_3 singlet representation, one in each family, without breaking the S_3 symmetry.

Hence, in order to impose S_3 as a fundamental symmetry, unbroken at the Fermi scale, we are led to extend the Higgs sector of the theory. The quark, lepton and Higgs fields are

$$\begin{aligned} Q^T &= (u_L, d_L), u_R, d_R, \\ L^T &= (\nu_L, e_L), e_R, \nu_R \quad \text{and } H, \end{aligned} \tag{4}$$

in an obvious notation. All of these fields have three species, and we assume that each one forms a reducible representation $\mathbf{1}_S \oplus \mathbf{2}$. The doublets carry capital indices I and J, which run from 1 to 2, and the singlets are denoted by $Q_3, u_{3R}, d_{3R}, L_3, e_{3R}, \nu_{3R}$ and H_S. Note that the subscript 3 denotes the singlet representation and not the third generation. The most general renormalizable Yukawa interactions of this model are given by

$$\mathcal{L}_Y = \mathcal{L}_{Y_D} + \mathcal{L}_{Y_U} + \mathcal{L}_{Y_E} + \mathcal{L}_{Y_\nu}, \tag{5}$$

where

$$\begin{aligned} \mathcal{L}_{Y_D} = &-Y_1^d \overline{Q}_I H_S d_{IR} - Y_3^d \overline{Q}_3 H_S d_{3R} - Y_2^d [\overline{Q}_I \kappa_{IJ} H_1 d_{JR} + \overline{Q}_I \eta_{IJ} H_2 d_{JR}] \\ &- Y_4^d \overline{Q}_3 H_I d_{IR} - Y_5^d \overline{Q}_I H_I d_{3R} + \text{h.c.}, \end{aligned} \tag{6}$$

$$\begin{aligned} \mathcal{L}_{Y_U} = &-Y_1^u \overline{Q}_I (\mathrm{i}\sigma_2) H_S^* u_{IR} - Y_3^u \overline{Q}_3 (\mathrm{i}\sigma_2) H_S^* u_{3R} - Y_2^u [\overline{Q}_I \kappa_{IJ} (\mathrm{i}\sigma_2) H_1^* u_{JR} \\ &+ \overline{Q}_I \eta_{IJ} (\mathrm{i}\sigma_2) H_2^* u_{JR}] - Y_4^u \overline{Q}_3 (\mathrm{i}\sigma_2) H_I^* u_{IR} - Y_5^u \overline{Q}_I (\mathrm{i}\sigma_2) H_I^* u_{3R} + \text{h.c.}, \end{aligned} \tag{7}$$

$$\begin{aligned} \mathcal{L}_{Y_E} = &-Y_1^e \overline{L}_I H_S e_{IR} - Y_3^e \overline{L}_3 H_S e_{3R} - Y_2^e [\overline{L}_I \kappa_{IJ} H_1 e_{JR} + \overline{L}_I \eta_{IJ} H_2 e_{JR}] \\ &- Y_4^e \overline{L}_3 H_I e_{IR} - Y_5^e \overline{L}_I H_I e_{3R} + \text{h.c.}, \end{aligned} \tag{8}$$

$$\mathcal{L}_{Y_\nu} = -Y_1^\nu \overline{L}_I (\mathrm{i}\sigma_2) H_S^* \nu_{IR} - Y_3^\nu \overline{L}_3 (\mathrm{i}\sigma_2) H_S^* \nu_{3R} - Y_2^\nu [\overline{L}_I \kappa_{IJ} (\mathrm{i}\sigma_2) H_1^* \nu_{JR} + \overline{L}_I \eta_{IJ} (\mathrm{i}\sigma_2) H_2^* \nu_{JR}] - Y_4^\nu \overline{L}_3 (\mathrm{i}\sigma_2) H_I^* \nu_{IR} - Y_5^\nu \overline{L}_I (\mathrm{i}\sigma_2) H_I^* \nu_{3R} + \text{h.c.}, \tag{9}$$

and

$$\kappa = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \text{and} \qquad \eta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{10}$$

Furthermore, we add to the Lagrangian the Majorana mass terms for the right-handed neutrinos:

$$\mathcal{L}_M = -M_1 \nu_{IR}^T C \nu_{IR} - M_3 \nu_{3R}^T C \nu_{3R}. \tag{11}$$

Due to the presence of three Higgs fields, the Higgs potential $V_H(H_S, H_D)$ is more complicated than that of the Standard Model. A Higgs potential invariant under S_3 was first proposed by Pakvasa and Sugawara [25], who assumed an additional reflection symmetry $R : H_s \rightarrow -H_s$. These authors found that in addition to the S_3 symmetry, their Higgs potential has an accidental permutational symmetry $S_2' : H_1 \leftrightarrow H_2$. The accidental S_2' symmetry is also present in our $V_H(H_S, H_D)$. The most general form of the potential $V_H(H_S, H_D)$ was investigated in detail by Kubo, Okada and Sakamaki [51], who discussed the potential of Pakvasa and Sugawara as a special case. A preliminary study on conditions under which the minimum of the Higgs potential is a global and stable one can be found in [52]. In this paper, we will assume that the vacuum respects the accidental S_2' symmetry of the Higgs potential and therefore that

$$\langle H_1 \rangle = \langle H_2 \rangle. \tag{12}$$

With these assumptions, the Yukawa interactions, equations (6)–(9) yield mass matrices, for all fermions in the theory, of the general form [20]

$$\mathbf{M} = \begin{pmatrix} \mu_1 + \mu_2 & \mu_2 & \mu_5 \\ \mu_2 & \mu_1 - \mu_2 & \mu_5 \\ \mu_4 & \mu_4 & \mu_3 \end{pmatrix}. \tag{13}$$

The Majorana mass for the left-handed neutrinos ν_L is generated by the see-saw mechanism. The corresponding mass matrix is given by

$$\mathbf{M}_\nu = \mathbf{M}_{\nu_\mathbf{D}} \tilde{\mathbf{M}}^{-1} \left(\mathbf{M}_{\nu_\mathbf{D}}\right)^T, \tag{14}$$

where $\tilde{\mathbf{M}} = \mathrm{diag}(M_1, M_1, M_3)$.

In principle, all entries in the mass matrices can be complex since there is no restriction coming from the flavour symmetry S_3. The mass matrices are diagonalized by bi-unitary transformations as

$$\begin{aligned} U_{d(u,e)L}^\dagger \mathbf{M}_{d(u,e)} U_{d(u,e)R} &= \mathrm{diag}(m_{d(u,e)}, m_{s(c,\mu)}, m_{b(t,\tau)}), \\ U_\nu^T \mathbf{M}_\nu U_\nu &= \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}). \end{aligned} \tag{15}$$

The entries in the diagonal matrices may be complex, so the physical masses are their absolute values.

The mixing matrices are, by definition,

$$V_{CKM} = U_{uL}^\dagger U_{dL}, \qquad V_{PMNS} = U_{eL}^\dagger U_\nu K. \tag{16}$$

where K is the diagonal matrix of the Majorana phase factors.

Table 1. Z_2 assignment in the leptonic sector.

$-$	$+$
H_S, ν_{3R}	$H_I, L_3, L_I, e_{3R}, e_{IR}, \nu_{IR}$

3. The mass matrices in the leptonic sector and Z_2 symmetry

A further reduction of the number of parameters in the leptonic sector may be achieved by means of an Abelian Z_2 symmetry. A possible set of charge assignments of Z_2, compatible with the experimental data on masses and mixings in the leptonic sector, is given in table 1.

These Z_2 assignments forbid the following Yukawa couplings:

$$Y_1^e = Y_3^e = Y_1^\nu = Y_5^\nu = 0. \tag{17}$$

Therefore, the corresponding entries in the mass matrices vanish, i.e., $\mu_1^e = \mu_3^e = 0$ and $\mu_1^\nu = \mu_5^\nu = 0$.

3.1. The mass matrix of the charged leptons

The mass matrix of the charged leptons takes the form

$$M_e = m_\tau \begin{pmatrix} \tilde{\mu}_2 & \tilde{\mu}_2 & \tilde{\mu}_5 \\ \tilde{\mu}_2 & -\tilde{\mu}_2 & \tilde{\mu}_5 \\ \tilde{\mu}_4 & \tilde{\mu}_4 & 0 \end{pmatrix}. \tag{18}$$

The unitary matrix U_{eL} that enters in the definition of the mixing matrix, V_{PMNS}, is calculated from

$$U_{eL}^\dagger M_e M_e^\dagger U_{eL} = \mathrm{diag}\big(m_e^2, m_\mu^2, m_\tau^2\big), \tag{19}$$

where m_e, m_μ and m_τ are the masses of the charged leptons [23]. The parameters $|\tilde{\mu}_2|$, $|\tilde{\mu}_4|$ and $|\tilde{\mu}_5|$ may readily be expressed in terms of the charged lepton masses [22]. The resulting expression for M_e, written to order $\big(m_\mu m_e/m_\tau^2\big)^2$ and $x^4 = (m_e/m_\mu)^4$, is

$$M_e \approx m_\tau \begin{pmatrix} \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & -\frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}}\,\mathrm{e}^{\mathrm{i}\delta_e} & \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}}\,\mathrm{e}^{\mathrm{i}\delta_e} & 0 \end{pmatrix}. \tag{20}$$

This approximation is numerically exact up to order 10^{-9} in units of the τ mass. Note that this matrix has no free parameters other than the Dirac phase δ_e.

The unitary matrix U_{eL} that diagonalizes $M_e M_e^\dagger$ and enters in the definition of the neutrino mixing matrix V_{PMNS} may be written as

$$U_{eL} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \mathrm{e}^{\mathrm{i}\delta_e} \end{pmatrix} \begin{pmatrix} O_{11} & -O_{12} & O_{13} \\ -O_{21} & O_{22} & O_{23} \\ -O_{31} & -O_{32} & O_{33} \end{pmatrix}, \tag{21}$$

where the orthogonal matrix $\mathbf{O}_{eL}$ on the right-hand side of equation (21), written to the same order of magnitude as M_e, is

$$\mathbf{O}_{eL} \approx \begin{pmatrix} \frac{1}{\sqrt{2}} x \frac{\left(1+2\tilde{m}_\mu^2+4x^2+\tilde{m}_\mu^4+2\tilde{m}_e^2\right)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} & -\frac{1}{\sqrt{2}} \frac{\left(1-2\tilde{m}_\mu^2+\tilde{m}_\mu^4-2\tilde{m}_e^2\right)}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} x \frac{\left(1+4x^2-\tilde{m}_\mu^4-2\tilde{m}_e^2\right)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} & \frac{1}{\sqrt{2}} \frac{\left(1-2\tilde{m}_\mu^2+\tilde{m}_\mu^4\right)}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{1}{\sqrt{2}} \\ -\frac{\sqrt{1+2x^2-\tilde{m}_\mu^2-\tilde{m}_e^2}\left(1+\tilde{m}_\mu^2+x^2-2\tilde{m}_e^2\right)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} & -x \frac{\left(1+x^2-\tilde{m}_\mu^2-2\tilde{m}_e^2\right)\sqrt{1+2x^2-\tilde{m}_\mu^2-\tilde{m}_e^2}}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{\sqrt{1+x^2}\tilde{m}_e\tilde{m}_\mu}{\sqrt{1+x^2-\tilde{m}_\mu^2}} \end{pmatrix}, \tag{22}$$

where, as before, $\tilde{m}_\mu = m_\mu/m_\tau$, $\tilde{m}_e = m_e/m_\tau$ and $x = m_e/m_\mu$.

3.2. The mass matrix of the neutrinos

According to the Z_2 selection rule, equation (17), the mass matrix of the Dirac neutrinos takes the form

$$\mathbf{M}_{\nu_\mathbf{D}} = \begin{pmatrix} \mu_2^\nu & \mu_2^\nu & 0 \\ \mu_2^\nu & -\mu_2^\nu & 0 \\ \mu_4^\nu & \mu_4^\nu & \mu_3^\nu \end{pmatrix}. \tag{23}$$

Then, the mass matrix for the left-handed Majorana neutrinos, $\mathbf{M}_\nu$, obtained from the see-saw mechanism, $\mathbf{M}_\nu = \mathbf{M}_{\nu_\mathbf{D}}\tilde{\mathbf{M}}^{-1}\left(\mathbf{M}_{\nu_\mathbf{D}}\right)^T$, is

$$\mathbf{M}_\nu = \begin{pmatrix} 2\left(\rho_2^\nu\right)^2 & 0 & 2\rho_2^\nu\rho_4^\nu \\ 0 & 2\left(\rho_2^\nu\right)^2 & 0 \\ 2\rho_2^\nu\rho_4^\nu & 0 & 2\left(\rho_4^\nu\right)^2 + \left(\rho_3^\nu\right)^2 \end{pmatrix}, \tag{24}$$

where $\rho_2^\nu = \left(\mu_2^\nu\right)/M_1^{1/2}$, $\rho_4^\nu = \left(\mu_4^\nu\right)/M_1^{1/2}$ and $\rho_3^\nu = \left(\mu_3^\nu\right)/M_3^{1/2}$; M_1 and M_3 are the masses of the right-handed neutrinos appearing in (11).

The non-Hermitian, complex, symmetric neutrino mass matrix M_ν may be brought to a diagonal form by a unitary transformation as

$$U_\nu^T M_\nu U_\nu = \mathrm{diag}\left(\left|m_{\nu_1}\right|\mathrm{e}^{\mathrm{i}\phi_1}, \left|m_{\nu_2}\right|\mathrm{e}^{\mathrm{i}\phi_2}, \left|m_{\nu_3}\right|\mathrm{e}^{\mathrm{i}\phi_\nu}\right), \tag{25}$$

where U_ν is the matrix that diagonalizes the matrix $M_\nu^\dagger M_\nu$.

As in the case of the charged leptons the matrices M_ν and U_ν can be reparametrized in terms of the complex neutrino masses. Then [22, 23]

$$M_\nu = \begin{pmatrix} m_{\nu_3} & 0 & \sqrt{(m_{\nu_3} - m_{\nu_1})(m_{\nu_2} - m_{\nu_3})}\,\mathrm{e}^{-\mathrm{i}\delta_\nu} \\ 0 & m_{\nu_3} & 0 \\ \sqrt{(m_{\nu_3} - m_{\nu_1})(m_{\nu_2} - m_{\nu_3})}\,\mathrm{e}^{-\mathrm{i}\delta_\nu} & 0 & (m_{\nu_1} + m_{\nu_2} - m_{\nu_3})\,\mathrm{e}^{-2\mathrm{i}\delta_\nu} \end{pmatrix} \tag{26}$$

and

$$U_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \mathrm{e}^{\mathrm{i}\delta_\nu} \end{pmatrix} \begin{pmatrix} \cos\eta & \sin\eta & 0 \\ 0 & 0 & 1 \\ -\sin\eta & \cos\eta & 0 \end{pmatrix}, \tag{27}$$

where

$$\sin^2\eta = \frac{m_{\nu_3} - m_{\nu_1}}{m_{\nu_2} - m_{\nu_1}}, \qquad \cos^2\eta = \frac{m_{\nu_2} - m_{\nu_3}}{m_{\nu_2} - m_{\nu_1}}. \tag{28}$$

The unitarity of U_ν constrains $\sin\eta$ to be real and thus $|\sin\eta| \leqslant 1$, this condition fixes the phases ϕ_1 and ϕ_2 as

$$|m_{\nu_1}|\sin\phi_1 = |m_{\nu_2}|\sin\phi_2 = |m_{\nu_3}|\sin\phi_\nu. \tag{29}$$

The only free parameters in these matrices are the phase ϕ_ν, implicit in m_{ν_1}, m_{ν_2} and m_{ν_3}, and the Dirac phase δ_ν.

3.3. The neutrino mixing matrix

The neutrino mixing matrix V_{PMNS} is the product $U_{eL}^{\dagger}U_\nu K$, where K is the diagonal matrix of the Majorana phase factors, defined by

$$\mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) = K^{\dagger}\mathrm{diag}(|m_{\nu_1}|, |m_{\nu_2}|, |m_{\nu_3}|)K^{\dagger}. \tag{30}$$

Except for an overall phase factor $\mathrm{e}^{\mathrm{i}\phi_1}$, which can be ignored, K is

$$K = \mathrm{diag}(1, \mathrm{e}^{\mathrm{i}\alpha}, \mathrm{e}^{\mathrm{i}\beta}), \tag{31}$$

where $\alpha = 1/2(\phi_1 - \phi_2)$ and $\beta = 1/2(\phi_1 - \phi_\nu)$ are the Majorana phases.

Therefore, the theoretical mixing matrix V_{PMNS}^{th} is given by

$$V_{PMNS}^{\mathrm{th}} = \begin{pmatrix} O_{11}\cos\eta + O_{31}\sin\eta\,\mathrm{e}^{\mathrm{i}\delta} & O_{11}\sin\eta - O_{31}\cos\eta\,\mathrm{e}^{\mathrm{i}\delta} & -O_{21} \\ -O_{12}\cos\eta + O_{32}\sin\eta\,\mathrm{e}^{\mathrm{i}\delta} & -O_{12}\sin\eta - O_{32}\cos\eta\,\mathrm{e}^{\mathrm{i}\delta} & O_{22} \\ O_{13}\cos\eta - O_{33}\sin\eta\,\mathrm{e}^{\mathrm{i}\delta} & O_{13}\sin\eta + O_{33}\cos\eta\,\mathrm{e}^{\mathrm{i}\delta} & O_{23} \end{pmatrix} \times K, \tag{32}$$

where $\cos\eta$ and $\sin\eta$ are given in equation (28), O_{ij} are given in equations (21) and (22), and $\delta = \delta_\nu - \delta_e$.

To find the relation of our results with the neutrino mixing angles we make use of the equality of the absolute values of the elements of V_{PMNS}^{th} and V_{PMNS}^{PDG} [53], that is

$$|V_{PMNS}^{\mathrm{th}}| = |V_{PMNS}^{PDG}|. \tag{33}$$

This relation allows us to derive expressions for the mixing angles in terms of the charged lepton and neutrino masses.

The magnitudes of the reactor and atmospheric mixing angles, θ_{13} and θ_{23}, are determined by the masses of the charged leptons only. Keeping only terms of order (m_e^2/m_μ^2) and $(m_\mu/m_\tau)^4$, we get

$$\sin\theta_{13} \approx \frac{1}{\sqrt{2}}x\frac{(1 + 4x^2 - \tilde{m}_\mu^4)}{\sqrt{1 + \tilde{m}_\mu^2 + 5x^2 - \tilde{m}_\mu^4}}, \qquad \sin\theta_{23} \approx \frac{1}{\sqrt{2}}\frac{1 + \frac{1}{4}x^2 - 2\tilde{m}_\mu^2 + \tilde{m}_\mu^4}{\sqrt{1 - 4\tilde{m}_\mu^2 + x^2 + 6\tilde{m}_\mu^4}}. \tag{34}$$

The magnitude of the solar angle depends on charged lepton and neutrino masses as well as the Dirac and Majorana phases

$$|\tan\theta_{12}|^2 = \frac{m_{\nu_2} - m_{\nu_3}}{m_{\nu_3} - m_{\nu_1}}\left(\frac{1 - 2\frac{O_{11}}{O_{31}}\cos\delta\sqrt{\frac{m_{\nu_3} - m_{\nu_1}}{m_{\nu_2} - m_{\nu_3}}} + \left(\frac{O_{11}}{O_{31}}\right)^2\frac{m_{\nu_3} - m_{\nu_1}}{m_{\nu_2} - m_{\nu_3}}}{1 + 2\frac{O_{11}}{O_{31}}\cos\delta\sqrt{\frac{m_{\nu_2} - m_{\nu_3}}{m_{\nu_3} - m_{\nu_1}}} + \left(\frac{O_{11}}{O_{31}}\right)^2\frac{m_{\nu_2} - m_{\nu_3}}{m_{\nu_3} - m_{\nu_1}}}\right). \tag{35}$$

The dependence of $\tan\theta_{12}$ on the Dirac phase δ, see (35), is very weak, since $O_{31} \sim 1$ but $O_{11} \sim 1/\sqrt{2}(m_e/m_\mu)$. Hence, we may neglect it when comparing (35) with the data on neutrino mixings.

The dependence of $\tan\theta_{12}$ on the phase ϕ_ν and the physical masses of the neutrinos enters through the ratio of the neutrino mass differences, it can be made explicit with the help of the unitarity constraint on U_ν, equation (29),

$$\frac{m_{\nu_2} - m_{\nu_3}}{m_{\nu_3} - m_{\nu_1}} = \frac{\left(|m_{\nu_2}|^2 - |m_{\nu_3}|^2 \sin^2\phi_\nu\right)^{1/2} - |m_{\nu_3}||\cos\phi_\nu|}{\left(|m_{\nu_1}|^2 - |m_{\nu_3}|^2 \sin^2\phi_\nu\right)^{1/2} + |m_{\nu_3}||\cos\phi_\nu|}. \tag{36}$$

Similarly, the Majorana phases are given by

$$\sin 2\alpha = \sin(\phi_1 - \phi_2) = \frac{|m_{\nu_3}|\sin\phi_\nu}{|m_{\nu_1}||m_{\nu_2}|} \times \left(\sqrt{|m_{\nu_2}|^2 - |m_{\nu_3}|^2\sin^2\phi_\nu} + \sqrt{|m_{\nu_1}|^2 - |m_{\nu_3}|^2\sin^2\phi_\nu}\right), \tag{37}$$

$$\sin 2\beta = \sin(\phi_1 - \phi_\nu) = \frac{\sin\phi_\nu}{|m_{\nu_1}|}\left(|m_{\nu_3}|\sqrt{1 - \sin^2\phi_\nu} + \sqrt{|m_{\nu_1}|^2 - |m_{\nu_3}|^2\sin^2\phi_\nu}\right). \tag{38}$$

A more complete and detailed discussion of the Majorana phases in the neutrino mixing matrix V_{PMNS} obtained in our model is given by Kubo [54].

4. Neutrino masses and mixings

In the present model, $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$ are determined by the masses of the charged leptons in very good agreement with the experimental values [11, 12, 55],

$$(\sin^2\theta_{13})^{\text{th}} = 1.1 \times 10^{-5}, \qquad (\sin^2\theta_{13})^{\text{exp}} \leqslant 0.046, \tag{39}$$

and

$$(\sin^2\theta_{23})^{\text{th}} = 0.499, \qquad (\sin^2\theta_{23})^{\text{exp}} = 0.5^{+0.06}_{-0.05}. \tag{40}$$

In this model, the experimental restriction $|\Delta m_{12}^2| < |\Delta m_{13}^2|$ implies an inverted neutrino mass spectrum, $|m_{\nu_3}| < |m_{\nu_1}| < |m_{\nu_2}|$ [20].

As can be seen from equations (35) and (36), the solar mixing angle is sensitive to the neutrino mass differences and the phase ϕ_ν but is only very weakly sensitive to the charged lepton masses. If we neglect the small terms proportional to O_{11} and O_{11}^2 in (35), we get

$$\tan^2\theta_{12} = \frac{\left(\Delta m_{12}^2 + \Delta m_{13}^2 + |m_{\nu_3}|^2\cos^2\phi_\nu\right)^{1/2} - |m_{\nu_3}||\cos\phi_\nu|}{\left(\Delta m_{13}^2 + |m_{\nu_3}|^2\cos^2\phi_\nu\right)^{1/2} + |m_{\nu_3}||\cos\phi_\nu|}. \tag{41}$$

From this expression, we may readily derive expressions for the neutrino masses in terms of $\tan\theta_{12}$ and ϕ_ν and the differences of the squared masses of the neutrinos

$$|m_{\nu_3}| = \frac{\sqrt{\Delta m_{13}^2}}{2\cos\phi_\nu \tan\theta_{12}} \frac{1 - \tan^4\theta_{12} + r^2}{\sqrt{1 + \tan^2\theta_{12}}\sqrt{1 + \tan^2\theta_{12} + r^2}}, \tag{42}$$

and

$$|m_{\nu_1}| = \sqrt{|m_{\nu_3}|^2 + \Delta m_{13}^2}, \qquad |m_{\nu_2}| = \sqrt{|m_{\nu_3}|^2 + \Delta m_{13}^2(1 + r^2)} \tag{43}$$

where $r^2 = \Delta m_{12}^2 / \Delta m_{13}^2 \approx 3 \times 10^{-2}$.

As $r^2 \ll 1$, the sum of the neutrino masses is

$$\sum_{i=1}^{3} \left|m_{\nu_i}\right| \approx \frac{\sqrt{\Delta m_{13}^2}}{2\cos\phi_\nu \tan\theta_{12}} \left(1 + 2\sqrt{1 + 2\tan^2\theta_{12}(2\cos^2\phi_\nu - 1) + \tan^4\theta_{12}} - \tan^2\theta_{12}\right). \tag{44}$$

The most restrictive cosmological upper bound for this sum is [17]

$$\sum |m_\nu| \leqslant 0.17 \text{ eV}. \tag{45}$$

From this upper bound and the experimentally determined values of $\tan\theta_{12}$ and Δm_{ij}^2, we may derive a lower bound for $\cos\phi_\nu$,

$$\cos\phi_\nu \geqslant 0.55, \tag{46}$$

or $0 \leqslant \phi_\nu \leqslant 57°$. The neutrino masses $|m_{\nu_i}|$ assume their minimal values when $\cos\phi_\nu = 1$. When $\cos\phi_\nu$ takes values in the range $0.55 \leqslant \cos\phi \leqslant 1$, the neutrino masses change very slowly with $\cos\phi_\nu$. In the absence of experimental information we will assume that ϕ_ν vanishes. Hence, setting $\phi_\nu = 0$ in our formula, we find

$$\left|m_{\nu_2}\right| \approx 0.056 \text{ eV} \qquad \left|m_{\nu_1}\right| \approx 0.055 \text{ eV} \qquad \left|m_{\nu_3}\right| \approx 0.022 \text{ eV}, \tag{47}$$

where we used the values $\Delta m_{13}^2 = 2.6 \times 10^{-3}$ eV^2, $\Delta m_{21}^2 = 7.9 \times 10^{-5}$ eV^2 and $\tan\theta_{12} = 0.667$ taken from [13].

5. Flavour-changing neutral currents (FCNC)

Models with more than one Higgs $SU(2)$ doublet have tree level flavour-changing neutral currents. In the minimal S_3-invariant extension of the Standard Model, considered here, there is one Higgs $SU(2)$ doublet per generation coupling to all fermions. The flavour-changing Yukawa couplings may be written in a flavour labelled symmetry adapted weak basis as

$$\begin{aligned} \mathcal{L}_Y^{\text{FCNC}} = {} & \left(\overline{E}_{aL} Y_{ab}^{ES} E_{bR} + \overline{U}_{aL} Y_{ab}^{US} U_{bR} + \overline{D}_{aL} Y_{ab}^{DS} D_{bR}\right) H_S^0 \\ & + \left(\overline{E}_{aL} Y_{ab}^{E1} E_{bR} + \overline{U}_{aL} Y_{ab}^{U1} U_{bR} + \overline{D}_{aL} Y_{ab}^{D1} D_{bR}\right) H_1^0 \\ & + \left(\overline{E}_{aL} Y_{ab}^{E2} E_{bR} + \overline{U}_{aL} Y_{ab}^{U2} U_{bR} + \overline{D}_{aL} Y_{ab}^{D2} D_{bR}\right) H_2^0 + \text{h.c.} \end{aligned} \tag{48}$$

where the entries in the column matrices $E's$, $U's$ and $D's$ are the left and right fermion fields and $Y_{ab}^{(e,u,d)s}$, $Y_{ab}^{(e,u,d)1,2}$ are 3×3 matrices of the Yukawa couplings of the fermion fields to the neutral Higgs fields H_s^0 and H_I^0 in the S_3-singlet and doublet representations, respectively.

In this basis, the Yukawa couplings of the Higgs fields to each family of fermions may be written in terms of matrices $\mathcal{M}_Y^{(e,u,d)}$, which give rise to the corresponding mass matrices $M^{(e,u,d)}$ when the gauge symmetry is spontaneously broken. From this relation we may calculate the flavour-changing Yukawa couplings in terms of the fermion masses and the vacuum expectation values of the neutral Higgs fields. For example, the matrix $\mathcal{M}_Y^e$ is written in terms of the matrices of the Yukawa couplings of the charged leptons as

$$\mathcal{M}_Y^e = Y_w^{E1} H_1^0 + Y_w^{E2} H_2^0, \tag{49}$$

in this expression the index w means that the Yukawa matrices are defined in the weak basis. The Yukawa couplings of immediate physical interest in the computation of the flavour-changing neutral currents are those defined in the mass basis, according to $\tilde{Y}_m^{EI} = U_{eL}^\dagger Y_w^{EI} U_{eR}$, where U_{eL} and U_{eR} are the matrices that diagonalize the charged lepton mass matrix defined in equations (15) and (21). We obtain [23]

$$\tilde{Y}_m^{E1} \approx \frac{m_\tau}{v_1} \begin{pmatrix} 2\tilde{m}_e & -\frac{1}{2}\tilde{m}_e & \frac{1}{2}x \\ -\tilde{m}_\mu & \frac{1}{2}\tilde{m}_\mu & -\frac{1}{2} \\ \frac{1}{2}\tilde{m}_\mu x^2 & -\frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \end{pmatrix}_m, \tag{50}$$

and

$$\tilde{Y}_m^{E2} \approx \frac{m_\tau}{v_2} \begin{pmatrix} -\tilde{m}_e & \frac{1}{2}\tilde{m}_e & -\frac{1}{2}x \\ \tilde{m}_\mu & \frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \\ -\frac{1}{2}\tilde{m}_\mu x^2 & \frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \end{pmatrix}_m, \tag{51}$$

where $\tilde{m}_\mu = 5.94 \times 10^{-2}$, $\tilde{m}_e = 2.876 \times 10^{-4}$ and $x = m_e/m_\mu = 4.84 \times 10^{-3}$. All the non-diagonal elements are responsible for tree-level FCNC processes. The actual values of the Yukawa couplings in equations (50) and (51) still depend on the VEVs of the Higgs fields v_1 and v_2 and hence on the Higgs potential. If the S_2' symmetry in the Higgs sector is preserved [25], $\langle H_1^0 \rangle = \langle H_2^0 \rangle = v$. In order to make an order of magnitude estimate of the coefficient in the Yukawa matrices, m_τ/v, we may further assume that the VEVs for all the Higgs fields are comparable, that is, $\tan\beta = \langle H_s^0 \rangle / \langle H_1^0 \rangle = 1$, and $\langle H_s^0 \rangle = \langle H_1^0 \rangle = \langle H_2^0 \rangle = \frac{\sqrt{2}}{\sqrt{3}} \frac{M_W}{g_2}$, then $m_\tau/v = \sqrt{3}/\sqrt{2} g_2 m_\tau / M_W$ and we may estimate the numerical values of the Yukawa couplings from the numerical values of the lepton masses. For instance, the amplitude of the flavour violating process $\tau^- \to \mu^- e^+ e^-$ is proportional to $\tilde{Y}_{\tau\mu}^E \tilde{Y}_{ee}^E$ [56]. Then, the leptonic branching ratio,

$$\mathrm{Br}(\tau \to \mu e^+ e^-) = \frac{\Gamma(\tau \to \mu e^+ e^-)}{\Gamma(\tau \to e\nu\bar{\nu}) + \Gamma(\tau \to \mu\nu\bar{\nu})} \tag{52}$$

and

$$\Gamma(\tau \to \mu e^+ e^-) \approx \frac{m_\tau^5}{3 \times 2^{10}\pi^3} \frac{\left(Y_{\tau\mu}^{1,2} Y_{ee}^{1,2}\right)^2}{M_{H_{1,2}}^4} \tag{53}$$

which is the dominant term, and the well-known expressions for $\Gamma(\tau \to e\nu\bar{\nu})$ and $\Gamma(\tau \to \mu\nu\bar{\nu})$ [53] give

$$\mathrm{Br}(\tau \to \mu e^+ e^-) \approx \frac{9}{4} \left(\frac{m_e m_\mu}{m_\tau^2}\right)^2 \left(\frac{m_\tau}{M_{H_{1,2}}}\right)^4, \tag{54}$$

taking for $M_{H_{1,2}} \sim 120$ GeV, we obtain

$$\mathrm{Br}(\tau \to \mu e^+ e^-) \approx 3.15 \times 10^{-17}$$

well below the experimental upper bound for this process, which is 2.7×10^{-7} [57]. Similar computations give the following estimates:

$$\mathrm{Br}(\tau \to e\gamma) \approx \frac{3\alpha}{8\pi} \left(\frac{m_\mu}{M_H}\right)^4, \tag{55}$$

$$\mathrm{Br}(\tau \to \mu\gamma) \approx \frac{3\alpha}{128\pi} \left(\frac{m_\mu}{m_\tau}\right)^2 \left(\frac{m_\tau}{M_H}\right)^4, \tag{56}$$

$$\mathrm{Br}(\tau \to 3\mu) \approx \frac{9}{64} \left(\frac{m_\mu}{M_H}\right)^4, \tag{57}$$

Table 2. Leptonic FCNC processes, calculated with $M_{H_{1,2}} \sim 120$ GeV.

FCNC processes	Theoretical BR	Experimental upper bound BR	References
$\tau \to 3\mu$	8.43×10^{-14}	2×10^{-7}	Aubert *et al* [57]
$\tau \to \mu e^+e^-$	3.15×10^{-17}	2.7×10^{-7}	Aubert *et al* [57]
$\tau \to \mu\gamma$	9.24×10^{-15}	6.8×10^{-8}	Aubert *et al* [58]
$\tau \to e\gamma$	5.22×10^{-16}	1.1×10^{-11}	Aubert *et al* [59]
$\mu \to 3e$	2.53×10^{-16}	1×10^{-12}	Bellgardt *et al* [60]
$\mu \to e\gamma$	2.42×10^{-20}	1.2×10^{-11}	Brooks *et al* [61]

$$\mathrm{Br}(\mu \to 3\mathrm{e}) \approx 18\left(\frac{m_e m_\mu}{m_\tau^2}\right)^2\left(\frac{m_\tau}{M_H}\right)^4, \tag{58}$$

and

$$\mathrm{Br}(\mu \to \mathrm{e}\gamma) \approx \frac{27\alpha}{64\pi}\left(\frac{m_e}{m_\mu}\right)^4\left(\frac{m_\tau}{M_H}\right)^4. \tag{59}$$

If we do not assume $v_s = v_1 = v_2$, but keep $v_s/v_1 = \tan\beta$ unspecified, the expressions (55)–(59) must be multiplied by a factor $(2+\tan^2\beta)^2/9$.

We see that FCNC processes in the leptonic sector are strongly suppressed by the small values of the mass ratios $m_e/m_\tau, m_\mu/m_\tau$ and m_τ/M_H. The numerical estimates of the branching ratios and the corresponding experimental upper bounds are shown in table 2. It may be seen that, in all cases considered, the numerical values for the branching ratios of the FCNC in the leptonic sector are well below the corresponding experimental upper bounds. The matrices of the quark Yukawa couplings may be computed in a similar way. Numerical values for the Yukawa couplings for u- and d-type quarks are given in our previous paper [20]. There, it was found that, due to the strong hierarchy in the quark masses and the corresponding small or very small mass ratios, the numerical values of all the Yukawa couplings in the quark sector are small or very small. Kubo, Okada and Sakamaki [51] have investigated the breaking of the gauge symmetry in the case of the most general Higgs potential invariant under S_3. They found that by breaking the S_3 symmetry very softly at very high energies it is possible to maintain the consistency and predictions of the present S_3-invariant extension of the Standard Model while simultaneously satisfying the experimental constraints for FCNC processes, that is, it is possible that all physical Higgs bosons, except one neutral one, could become sufficiently heavy ($M_H \sim 10$ TeV) to suppress all the flavour-changing neutral current processes in the quark sector of the theory without having a problem with triviality.

6. Muon anomalous magnetic moment

In models with more than one Higgs $SU(2)$ doublet, the exchange of flavour-changing scalars may contribute to the anomalous magnetic moment of the muon. In the minimal S_3-invariant extension of the Standard Model we are considering here, we have three Higgs $SU(2)$ doublets, one in the singlet and the other two in the doublet representations of the S_3 flavour group. The Z_2 symmetry decouples the charged leptons from the Higgs boson in the S_3 singlet representation. Therefore, in the theory there are two neutral scalars and two neutral pseudoscalars whose exchange will contribute to the anomalous magnetic moment of the muon, in the leading order of magnitude. Since the heavier generations have larger

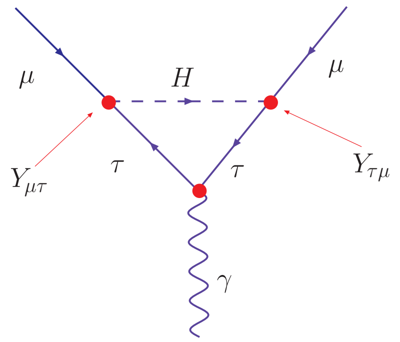

Figure 1. The contribution, $\delta a_\mu^{(H)}$, to the anomalous magnetic moment of the muon from the exchange of flavour-changing scalars. The neutral Higgs boson can be a scalar or a pseudoscalar. (This figure is in colour only in the electronic version)

flavour-changing couplings, the largest contribution comes from the heaviest charged leptons coupled to the lightest of the neutral Higgs bosons, $\mu - \tau - H$, as shown in figure 1.

A straightforward computation gives

$$\delta a_\mu^{(H)} = \frac{Y_{\mu\tau} Y_{\tau\mu}}{16\pi^2} \frac{m_\mu m_\tau}{M_H^2} \left(\log\left(\frac{M_H^2}{m_\tau^2} \right) - \frac{3}{2} \right). \tag{60}$$

With the help of (50) and (51) we may write $\delta a_\mu^{(H)}$ as

$$\delta a_\mu^{(H)} = \frac{m_\tau^2}{(246\ \mathrm{GeV})^2} \frac{(2 + \tan^2\beta)}{32\pi^2} \frac{m_\mu^2}{M_H^2} \left(\log\left(\frac{M_H^2}{m_\tau^2} \right) - \frac{3}{2} \right), \tag{61}$$

in this expression $\tan\beta = v_s/v_1$ is the ratio of the vacuum expectation values of the Higgs scalars in the singlet representation, v_s, and in the doublet representation, v_1, of the S_3 flavour group. The most restrictive upper bound on $\tan\beta$ may be obtained from the experimental upper bound on $\mathrm{Br}(\mu \to 3e)$ given in (58), and in table 2 we obtain

$$\tan\beta \leqslant 14. \tag{62}$$

Substitution of this value into (61) and taking for the Higgs mass the value $M_H = 120$ GeV gives an estimate of the largest possible contribution of the FCNC to the anomaly of the muon's magnetic moment:

$$\delta a_\mu^{(H)} \approx 1.7 \times 10^{-10}. \tag{63}$$

This number has to be compared with the difference between the experimental value and the Standard Model prediction for the anomaly of the muon's magnetic moment [62],

$$\Delta a_\mu = a_\mu^{\exp} - a_\mu^{SM} = (28.7 \pm 9.1) \times 10^{-10}, \tag{64}$$

which means

$$\frac{\delta a_\mu^{(H)}}{\Delta a_\mu} \approx 0.06. \tag{65}$$

Hence, the contribution of the flavour-changing neutral currents to the anomaly of the muon's magnetic moment is smaller than or of the order of 6% of the discrepancy between the experimental value and the Standard Model prediction. This discrepancy is of the order of three standard deviations and quite important, but its interpretation is compromised by uncertainties in the computation of higher order hadronic effects arising mainly from three-loop vacuum polarization effects, $a_\mu^{\rm VP}(3,{\rm had}) \approx -1.82 \times 10^{-9}$ [63], and from three-loop contributions of hadronic light by light type, $a_\mu^{\rm LBL}(3,{\rm had}) \approx 1.59 \times 10^{-9}$ [63]. As explained above, the contribution to the anomaly from flavour-changing neutral currents in the minimal S_3-invariant extension of the Standard Model, computed in this work, is, at most, 6% of the discrepancy between the experimental value and the Standard Model prediction for the anomaly and is of the same order of magnitude as the uncertainties in the higher order hadronic contributions, but still it is not negligible and is certainly compatible with the best, state-of-the-art, experimental measurements and theoretical computations.

7. Conclusions

In the minimal S_3-invariant extension of the SM the flavour symmetry group $Z_2 \times S_3$ relates the mass spectrum and mixings. This allowed us to compute the neutrino mixing matrix explicitly in terms of the masses of the charged leptons and neutrinos [22]. In this model, the magnitudes of the three mixing angles are determined by the interplay of the flavour $S_3 \times Z_2$ symmetry, the see-saw mechanism and the lepton mass hierarchy. We also found that V_{PMNS} has three CP violating phases, one Dirac phase $\delta = \delta_\nu - \delta_e$ and two Majorana phases, α and β, that are functions of the neutrino masses and another phase ϕ_ν which is independent of the Dirac phase. The numerical values of the reactor, θ_{13}, and the atmospheric, θ_{23}, mixing angles are determined by the masses of the charged leptons only, in very good agreement with the experiment. The solar mixing angle θ_{12} is almost insensitive to the values of the masses of the charged leptons, but its experimental value allowed us to fix the scale and origin of the neutrino mass spectrum, which has an inverted hierarchy, with the values $|m_{\nu_2}| = 0.056$ eV, $|m_{\nu_1}| = 0.055$ eV and $|m_{\nu_3}| = 0.022$ eV. We also obtained explicit expressions for the matrices of the Yukawa couplings of the lepton sector parametrized in terms of the charged lepton masses and the VEVs of the neutral Higgs bosons in the S_3-doublet representation. These Yukawa matrices are closely related to the fermion mass matrices and have a structure of small and very small entries reflecting the observed charged lepton mass hierarchy. With the help of the Yukawa matrices, we computed the branching ratios of a number of FCNC processes and found that the branching ratios of all FCNC processes considered are strongly suppressed by powers of the small mass ratios m_e/m_τ and m_μ/m_τ and by the ratio $\left(m_\tau/M_{H_{1,2}}\right)^4$, where $M_{H_{1,2}}$ is the mass of the neutral Higgs bosons in the S_3-doublet. Taking for $M_{H_{1,2}}$ a very conservative value ($M_{H_{1,2}} \approx 120$ GeV), we found that the numerical values of the branching ratios of the FCNC in the leptonic sector are well below the corresponding experimental upper bounds by many orders of magnitude. It has already been argued that small FCNC processes mediating non-standard quark–neutrino interactions could be important in the theoretical description of the gravitational core collapse and shock generation in the explosion stage of a supernova [64–66]. Finally, the contribution of the flavour-changing neutral currents to the anomalous magnetic moment of the muon is small but non-negligible and it is compatible with the best state-of-the-art measurements and theoretical computations.

Acknowledgments

We thank Professor Jens Erler and Dr Genaro Toledo-Sanchez for helpful discussions about $g-2$. This work was partially supported by CONACYT México under contract no 51554-F and by DGAPA-UNAM under contract PAPIIT-IN115207-2.

References

[1] Jung C K, Grew C Mc, Kajita T and Mann T 2001 *Annu. Rev. Nucl. Part. Sci* **51** 451
[2] Mohapatra R N and Smirnov A Y 2006 *Ann. Rev. Nucl. Part. Sci.* **56** 569 (*Preprint* hep-ph/0603118)
[3] Altmann M *et al* (GNO Collaboration) 2005 *Phys. Lett.* B **616** 174
[4] Smy M B *et al* (SK Collaboration) 2004 *Phys. Rev.* D **69** 011104
[5] Ahmad Q R *et al* (SNO Collaboration) 2002 *Phys. Rev. Lett.* **89** 011301
[6] Aharmim B *et al* (SNO Collaboration) 2005 *Phys. Rev.* C **72** 055502 (*Preprint* nucl-ex/0502021)
[7] Fukuda S *et al* (SK Collaboration) 2002 *Phys. Lett.* B **539** 179
[8] Ashie Y *et al* 2004 *Phys. Rev. Lett.* **93** 101801 (*Preprint* hep-ex/0404034)
[9] Bemporad C, Gratta G and Vogel P 2002 *Rev. Mod. Phys.* **74** 297
[10] Araki T *et al* (KamLAND Collaboration) 2005 *Phys. Rev. Lett.* **94** 081801
[11] Maltoni M, Schwetz T, Tórtola M A and Valle J W F 2004 *New J. Phys.* **6** 122
[12] Schwetz T 2005 Neutrino oscillations: current status and prospects *Acta Phys. Polon.* B **36** 3203 (*Preprint* hep-ph/0510331)
[13] Gonzalez-Garcia M C and Maltoni M 2007 *Preprint* hep-ph/0704.1800
[14] Apollonio M *et al* (CHOOZ Collaboration) 2003 *Eur. Phys. J.* C **27** 331
[15] Eitel K 2004 *'Neutrino 2004' 21st Int. Conf. on Neutrino Physics and Astrophysics (Paris, France 2004)* ed J Dumarchey, Th Patyak and F Vanuucci (2005 *Nucl. Phys.* B (*Proc. Suppl.*) **143** 197)
[16] Eliot S R and Engel J 2004 *J. Phys. G: Nucl. Part. Phys.* **30** R183
[17] Seljak U, Slosar A and McDonald P 2006 *J. Cosmol. Astropart. Phys.* JCAP10(2006)014 (*Preprint* astro-ph/0604335)
[18] Elgaroy O and Lahav O 2005 *New J. Phys.* **7** 61
[19] Lesgourgues J and Pastor S 2006 *Phys. Rep.* **429** 307 (*Preprint* astro-ph/0603494)
[20] Kubo J, Mondragón A, Mondragón M and Rodríguez-Jáuregui E 2003 *Prog. Theor. Phys.* **109** 795
Kubo J, Mondragón A, Mondragón M and Rodríguez-Jáuregui E 2005 *Prog. Theor. Phys.* **114** 287 (erratum)
[21] Kubo J, Mondragon A, Mondragon M, Rodriguez-Jauregui E, Felix-Beltran O and Peinado E 2005 *J. Phys. Conf. Ser.* **18** 380
[22] Felix O, Mondragon A, Mondragon M and Peinado E 2006 *Rev. Mex. Fis.* S **52N4** 67 (*Preprint* hep-ph/0610061)
[23] Mondragon A, Mondragon M and Peinado E 2007 *Phys. Rev.* D **76** 076003 (*Preprint* hep-ph/0706.0354)
[24] Fritzsch H 1977 *Phys. Lett.* B **70** 436
[25] Pakvasa S and Sugawara H 1978 *Phys. Lett.* **73B** 61
[26] Fritzsch H 1978 *Phys. Lett.* B **73** 317
[27] Harari H, Haut H and Weyers J 1978 *Phys. Lett.* B **78** 459
[28] Frere J M 1979 *Phys. Lett.* B **80** 369
[29] Fritzsch H 1979 *Nucl. Phys.* B **155** 189
[30] Yamanaka Y, Pakvasa S and Sugawara H 1982 *Phys. Rev.* D **25** 1895
Yamanaka Y, Pakvasa S and Sugawara H 1984 *Phys. Rev.* D **29** 2135 (erratum)
[31] Kaus P and Meshkov S 1990 *Phys. Rev.* D **42** 1863
[32] Fritzsch H and Plankl J P 1990 *Phys. Lett.* B **237** 451
[33] Harrison P F and Scott W G 1994 *Phys. Lett.* B **333** 471
[34] Mondragón A and Rodróiguez-Jíauregui E 1999 *Phys. Rev.* D **59** 093009
[35] Mondragón A and Rodríguez-Jáuregui E 2000 *Phys. Rev.* D **61** 113002
[36] For a review see Fritzsch H and Xing Z Z 2000 *Prog. Part. Nucl. Phys.* **45** 1
[37] Hall L J and Murayama H 1995 *Phys. Rev. Lett.* **75** 3985
[38] Carone C D, Hall L J and Murayama H 1996 *Phys. Rev.* D **53** 6282
[39] Koide Y 1999 *Phys. Rev.* D **60** 077301
[40] Ma E 1991 *Phys. Rev.* D **44** 587
[41] Ma E 2002 *Mod. Phys. Lett.* A **17** 627
Ma E 2002 *Mod. Phys. Lett.* A **17** 2361
[42] Babu K S, Ma E and Valle J W F 2003 *Phys. Lett.* B **552** 207
[43] Chen S-L, Frigerio M and Ma E 2004 *Phys. Rev.* D **70** 073008
Chen S-L, Frigerio M and Ma E 2004 *Phys. Rev.* D **70** 079905 (erratum)
[44] Grimus W and Lavoura L 2005 *J. High Energy Phys.* JHEP08(2005)013
[45] Hagedorn C, Lindner M and Mohapatra R N 2006 *J. High Energy Phys.* JHEP06(2006)042 (*Preprint* hep-ph/0602244)
[46] Hagedorn C, Lindner M and Plentinger F 2006 *Phys. Rev.* D **74** 025007 (*Preprint* hep-ph/0604265)
[47] Smirnov A Y 2006 *J. Phys. Conf. Ser.* **53** 44

[48] Altarelli G and Feruglio F 2004 *New J. Phys.* **6** 106
Altarelli G 2006 *Proc. IPM School and Conf. on Lepton and Hadron Physics (IPM-LHP06) (Tehran, Iran, 15–20 May 2006)* p 0001 (*Preprint* hep-ph/0610164)
[49] Mondragon A 2006 *AIP Conf. Ser.* **857B** 266–82 (*Preprint* hep-ph/0609243)
[50] Albright C H and Chen M C 2006 *Phys. Rev.* D **74** 113006 (*Preprint* hep-ph/0608137)
[51] Kubo J, Okada H and Sakamaki F 2004 *Phys. Rev.* D **70** 036007
[52] Emmanuel-Costa D, Felix-Beltran O, Mondragon M and Rodriguez-Jauregui E 2007 *AIP Conf. Proc.* **917** 390
[53] Yao W-M *et al* (Particle Data Group) 2006 *J. Phys. G: Nucl. Part. Phys.* **33** 1
[54] Kubo J 2004 *Phys. Lett.* B **578** 156
Kubo J 2005 *Phys. Lett.* **619** 387 (erratum)
[55] Fogli G L, Lisi E, Marrone A and Palazzo A 2006 *Prog. Part. Nucl. Phys.* **57** 742–95 (*Preprint* hep-ph/0506083)
[56] Sher M and Yuan Y 1991 *Phys. Rev.* D **44** 1461
[57] Aubert B *et al* (BABAR Collaboration) 2004 *Phys. Rev. Lett.* **92** 121801
[58] Aubert B *et al* (BABAR Collaboration) 2005 *Phys. Rev. Lett.* **95** 041802
[59] Aubert B *et al* (BABAR Collaboration) 2006 *Phys. Rev. Lett.* **96** 041801
[60] Bellgardt U *et al* (SINDRUM Collaboration) 1998 *Nucl. Phys.* B **299** 1
[61] Brooks M L *et al* (MEGA Collaboration) 1999 *Phys. Rev. Lett.* **83** 1521 (*Preprint* hep-ex/9905013)
[62] Jegerlehner F 2007 *Acta Phys. Polon.* B **38** 3021 (*Preprint* hep-ph/0703125)
[63] Erler J and Sanchez G T 2006 *Phys. Rev. Lett.* **97** 161801 (*Preprint* hep-ph/0605052)
[64] Raffelt G G 2006 Supernova neutrino observations: what can we learn? *Proc. 22nd Int. Conf. Neutrino Physics and Astrophysics (Neutrino 2006) (Santa Fe, New Mexico, 13–19 June 2006)* (*Preprint* astro-ph/0701677)
[65] Amanik P S, Fuller G M and Grinstein B 2005 *Astropart. Phys.* **24** 160 (*Preprint* hep-ph/0407130)
[66] Esteban-Pretel A, Tomas R and Valle J W F 2007 Probing non-standard neutrino interactions with supernova neutrinos *Preprint* arXiv:0704.0032